

CHARACTERISTICS OF A NEW OSCILLATING-PLATE VISCOMETER

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Synopsis : The characteristics of a new oscillating-plate viscometer, constructed by Sasahara et al., have been investigated experimentally over a wide range of viscosity. In particular, the following were focused on: 1) the changes in the resonant frequency from air into liquid. 2) the effects of both the wall of a vessel and the thickness of the oscillating-plate. It is shown that the working formula for viscosity determination by the oscillating-plate method should be corrected by considering the above two effects.

Key words : viscosity, oscillating-plate viscometer.

1. Introduction

A new oscillating-plate viscometer has been constructed by Sasahara et al.⁽¹⁾ to measure the viscosities of molten slags and fluxes with high accuracy. This type of viscometer provides instantaneous and continuous data for the product of density and viscosity of the melts; these are recorded automatically by measuring the amplitudes of the plate oscillation. In this viscometer, a leaf-spring is equipped for chinning the plate attached to stem and a laser displacement transducer is used for determining accurate amplitudes. The equation used to connect the measured amplitude with the viscosity of liquid in the oscillating-plate method has been derived on the basis of several assumptions as will be described later. The experimental conditions of the apparatus, however, do not satisfy all of those assumptions. The characteristics of the viscometer, therefore, must be elucidated to obtain accurate viscosity values of molten slags and fluxes. The purpose of the present work is to study experimentally the characteristics of the oscillating-plate viscometer over a wide range of viscosity i.e. 1~100 Pa·s, which covers the viscosity range of molten slags and fluxes.

2. Principle of viscosity measurement by the oscillating-plate method

When a flat plate is now vibrating in a liquid with a constant driving force, the amplitude of motion of the plate is reduced to a degree dependent on the viscosity of

the liquid. The oscillating-plate method is based on measurements of the amplitudes of plate oscillations in air and in a liquid sample. The relation between viscosity and amplitudes has been derived theoretically as follows^{2, 3)}:

$$\rho \mu = \frac{R_M^2}{\pi f A^2} \cdot \left(\frac{f_a \cdot E_a}{f \cdot E} - 1 \right)^2 = K \cdot \theta_o \quad \dots(1)$$

$$K = \frac{R_M^2}{\pi f_a A^2} \quad , \text{and} \quad \theta_o = \frac{f_a}{f} \cdot \left(\frac{f_a \cdot E_a}{f \cdot E} - 1 \right)^2$$

where ρ is the liquid density, μ the liquid viscosity, E and E_a the amplitudes in liquid and in air, respectively, f and f_a the resonant frequencies in liquid and in air, respectively, K a constant of the apparatus, R_M the real component of the mechanical impedance, A the area of the plate.

Equation (1) has been based on the following assumptions⁴⁾:

- (1) A liquid sample is a Newtonian fluid.
- (2) The plate oscillation does not make turbulent flow.
- (3) An oscillating-plate does not slip on the contact surface of a liquid sample.
- (4) The size of an oscillating-plate is larger than one wavelength of the wave, which is produced by the oscillation. The wave must be a plane wave.
- (5) The end effect of an oscillating-plate on the damping of the amplitude is negligible.
- (6) The size of a vessel is large enough to neglect the effect of reflected wave from the wall of the vessel.

If the resonant frequency in liquid f is assumed to be equal to that in air f_a in Eq.(1), the following equation is obtained :

$$\rho \mu = K \left(\frac{E_a}{E} - 1 \right)^2 = K \cdot \theta \quad \dots(2)$$

where $K = \frac{R_M^2}{\pi f_a A^2}$, and $\theta = \left(\frac{E_a}{E} - 1 \right)^2$

Hereafter, θ is called the damping-factor. The constant of the apparatus K is determined experimentally, previous to viscosity measurements, by the use of Standard Viscosity Samples at room temperature.

3. Viscosity Apparatus

Figure 1 shows the schematic diagram of the oscillating-plate viscometer, which comprises an oscillator, an oscillating-plate, a displacement transducer and a computer. It would be expected that the highly accurate oscillation can be obtained by the use of the leaf-spring, which has been developed by Sasahara et al¹⁾. The stem connecting the oscillating-plate is chinned from the center of the leaf-spring. The displacements of the target attached to the stem, i.e., the amplitudes of the plate oscillations, are

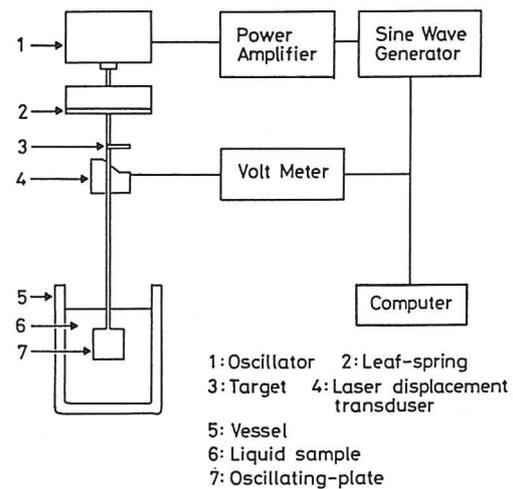


Fig.1 Schematic diagram of the oscillating-plate viscometer.

measured by the laser displacement transducer, and recorded in a computer. In these experiments, square oscillating-plates (length of side, 30 mm) are employed.

4. Characteristics of the oscillating-plate viscometer

4.1. Dependence of the resonant frequency on $\rho\mu$ value

Equation (2) can be obtained on the assumption that the resonant frequency f in liquid is equal to f_a in air. Figure 2 shows the experimental results of the relation between the resonant frequencies f and $\rho\mu$ values for the present oscillating-plate viscometer with the leaf-spring. The liquids used for the experiments are Standard Viscosity Samples of 5 kinds of viscosities. The distance between the oscillating-plate and the wall in the vessel is kept to be 40 mm. The result obtained in an oscillating-plate viscometer with a sound speaker as an oscillator³⁾ is also indicated in this figure. As shown in Fig.2, the dependence of the resonant frequency on $\rho\mu$ value is negligibly small for the viscosity range of $250 \sim 35000 \text{ kg}^2\text{m}^{-4}\text{s}^{-1}$ ($\cong 0.25 \sim 35 \text{ Pa}\cdot\text{s}$) when the leaf-spring of 0.5 or 0.6 mm thickness is used in the present apparatus.

4.2. Effect of reflected wave from the wall of a vessel

The effect of reflected wave from the wall, which is produced by the plate oscillations and travels towards the direction perpendicular to the plate, has been studied experimentally for liquids over a viscosity range of $10^3 \sim 10^5 \text{ kg}^2\text{m}^{-4}\text{s}^{-1}$ ($\cong 1 \sim 100 \text{ Pa}\cdot\text{s}$) in a rectangular vessel as shown in Fig.3. Silicone-oil with 5 kinds of viscosities is employed as the liquid samples. The leaf-spring of 0.6 mm thickness is used for the experiments. Figure 4 shows the plots of $\log(\theta)$ against the distance between the plate and the wall for various $\rho\mu$ values; where the solid curve indicates one wavelength λ of the wave produced by the oscillations for each $\rho\mu$ value. As shown in this figure, the effect

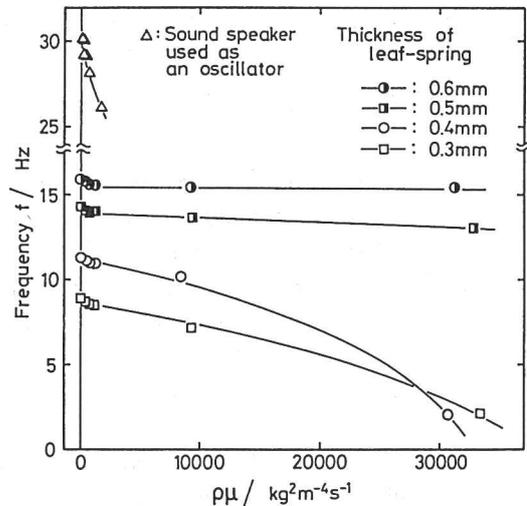


Fig.2 Dependence of the resonant frequency on $\rho\mu$ value.

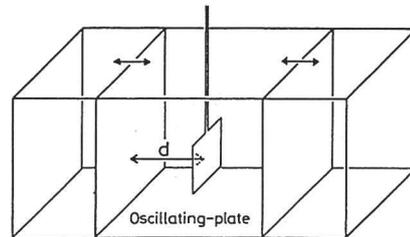


Fig.3 Schematic diagram of a vessel with variable walls.

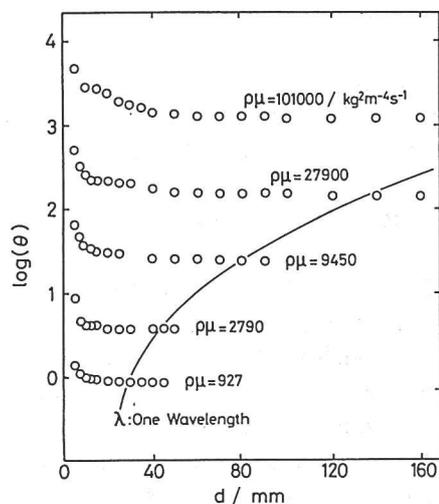


Fig.4 Plots of $\log(\theta)$ against the distance for various $\rho\mu$ values.

of reflected wave increases with decreasing the distance between the plate and the wall. In addition, the value of $\log(\theta)$ at the distance of one wavelength λ is almost the same as that at infinity. In other words, the effect of reflected wave can be neglected for a viscosity range of $10^3 \sim 10^5 \text{ kg}^2\text{m}^{-4}\text{s}^{-1}$ ($\approx 1 \sim 100 \text{ Pa}\cdot\text{s}$) when the distance between the plate and the wall is kept over one wavelength.

4.3. End effect of the oscillating-plate

Equations (1) and (2) have been derived on the assumption that the thickness of the plate is infinitely thin. Since the thickness of the plate is actually finite in any viscometer, the end effect of the oscillating-plate should not be neglected. If we now assume that the effect of the thickness of the plate corresponds to the increase ΔA_E in the area of the plate and furthermore ΔA_E is proportional to the thickness of the plate T , the following equation is obtained from Eq.(2) :

$$\begin{aligned} \frac{\theta}{\rho \mu} &= \frac{\pi f_a}{R_M^2} (A + \Delta A_E)^2 = \frac{\pi f_a}{R_M^2} (A + C_E \cdot T)^2 \\ &= a + b \cdot T \end{aligned} \quad \dots(3)$$

$$\text{where } a = \frac{\pi f_a A^2}{R_M^2}, \text{ and } b = \frac{2 \pi f_a A C_E}{R_M^2}$$

In the above equation, since ΔA_E is considered to be much smaller than the area of the plate A , the term $(\Delta A_E)^2$ can be neglected.

The relation between the damping-factor θ and the thickness T has been determined experimentally. Square oscillating-plates of polyvinyl chloride (length of side, 30 mm; thickness, 2.15, 3.10, 3.90 and 5.00 mm) are used for the experiments. The leaf-spring of 0.6 mm thickness is equipped for chinning the plates. The distance between the plate and the wall of the vessel is separated at 40 mm. The relation between $\theta/\rho\mu$ and T for various $\rho\mu$ values is shown in Fig.5, where the linear relation is obtained as expected from Eq.(3). It is obvious from this figure that the end effect of the oscillating-plate increases with increasing not only the thickness of the plate but also the viscosity of the liquid samples.

5. Empirical relation between the damping factor θ and $\rho\mu$ values

Equation (2) may be written in the form

$$\log(\rho \mu) = \log K + \log(\theta) \quad \dots(4)$$

Figure 6 shows the relation between $\log(\rho\mu)$ and $\log(\theta)$ for various thicknesses of the oscillating-plates. As can be seen from Fig.6, however, the relation between $\log(\rho\mu)$ and $\log(\theta)$, i.e., a linear relation with the slope of 45 degree given by

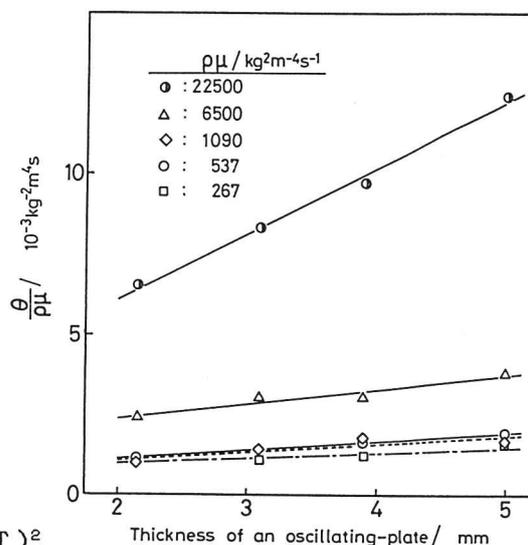


Fig.5 Effect of the thickness of the oscillating-plate (End effect).

Eq.(4), does not, to a good approximation, hold because of the effect of reflected wave from the wall of a vessel and the end effect of the oscillating-plates. Therefore, an empirical relation between $\log(\rho \mu)$ and $\log(\theta)$ should be determined so as to best fit the experimental results indicated in Fig.6. For example, the relation between $\log(\rho \mu)$ and $\log(\theta)$ for the triangles plotted in Fig.6 can be represented as a function of $\log(\theta)$ in the following form :

$$\log(\rho \mu) = 3.03 + 8.17 \times 10^{-1} \log(\theta) - 7.53 \times 10^{-2} \{\log(\theta)\}^2 \dots (5)$$

In Fig.7, the above relation of the second regression is shown together with that of the first order regression for the five points of triangles. The experimentally determined relation in the form of Eq.(5) can be used for determining accurate viscosities of molten slags and fluxes at high temperatures.

6. Application of the oscillating-plate method to the measurements for the viscosity of the standard reference material at high temperatures

Viscosity of the standard reference material (SRM)⁵⁾, which is provided from Community Bureau of Reference in Europe for the establishment of standard viscosity samples for high temperature viscosity measurements, has been measured by the oscillating-plate method according to the protocol of the project in Phase 2⁵⁾. The composition of the material is shown in Table 1. Figure 8 shows the experimental results of temperature dependence of $\log(\mu)$ with the results reported by various laboratories. This figure includes the values of $\log(\mu)$ obtained from the second order regression of $\log(\rho \mu)$ vs. $\log(\theta)$, together with those of the first order regression. Since the viscosity range of SRM corresponds to

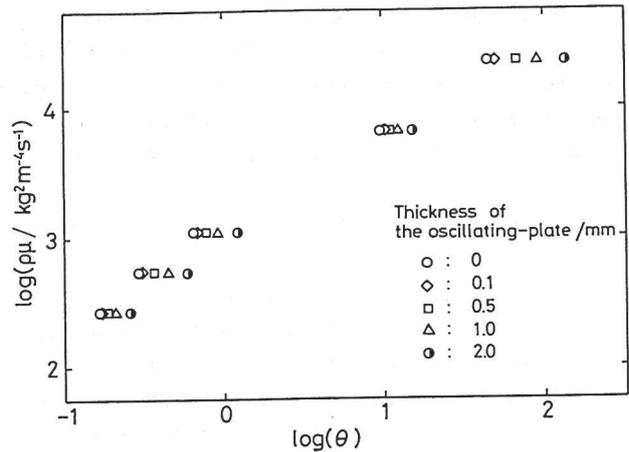


Fig.6 Plots of $\log(\rho \mu)$ against $\log(\theta)$ for various thicknesses of oscillating-plates.

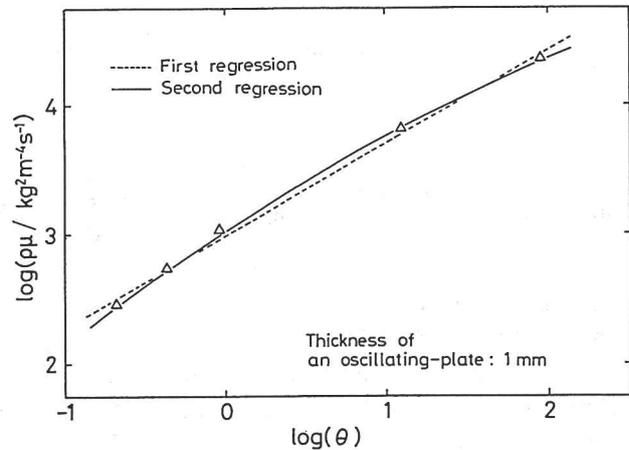


Fig.7 Plots of $\log(\rho \mu)$ against $\log(\theta)$.

Table 1 Chemical composition(mass%) of SRM.

SiO ₂	Li ₂ O	Al ₂ O ₃		
63.5	21.7	14.8		
Na ₂ O	K ₂ O	C	Fe ₂ O ₃	CaO
0.3	0.1	0.1	0.1	0.1

$\log(\rho \mu) = 2 \sim 4$ as shown in Fig.7, the results obtained from the second regression are higher than those from the first order regression. The empirical relation between $\log(\rho \mu)$ and $\log(\theta)$, in which the effect of reflected wave from the wall of the vessel and the end effect of the plate are included, should be used for accurate and reliable viscosity measurements.

7. Conclusions

The characteristics of the oscillating-plate viscometer, constructed by Sasahara et al.⁽¹⁾, have been investigated experimentally. The results obtained are as follows:

- (1) The resonant frequency of the plate oscillation in a liquid sample decreases with increasing $\rho \mu$ values. However, the viscosity dependence of the resonant frequency is negligibly small when an adequate leaf-spring is used for chinning the plate attached to stem.
- (2) The effect of reflected wave can be neglected when the distance between the plate and the wall of a vessel is kept over one wavelength produced by the plate oscillation.
- (3) The end effect of the oscillating-plate increases with increasing not only the thickness of the plate but also the viscosity of a liquid.
- (4) Owing to the above two effects, the working equation should be corrected experimentally for accurate viscosity determinations of molten slags and fluxes by the oscillating-plate method.

Acknowledgments

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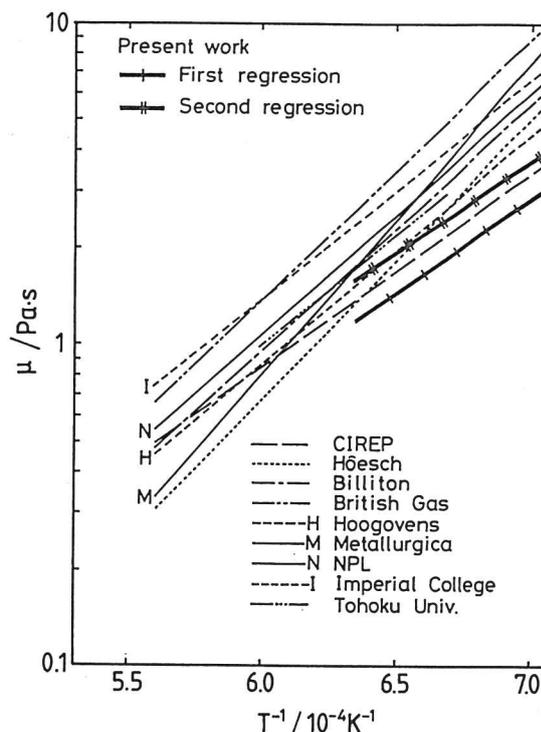


Fig.8 Viscosity of SRM as a function of the reciprocal of the temperature.